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Transfinite harmonization by taking the dissonance out of the quantum field symphony

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Abstract

Particle physics may be likened to a magnificent symphony. Alas due to some instrumental defects, wrong reading of the notes and a lack of virtuosity of some members of the orchestra, a non-negligible number of dissonants are making it sound less than perfect. By means of the specific example of renormalization groups applied to GUT unification, the present work aims at illustrating the point we just made and showing how a simplictic transfinite adjustment of our formulas lead to harmonization and consequently considerable simplification of well known theories which goes as far as facilitating the discovery of new connections and the solution of many problems which were previously thought very hard, if at all possible, to solve.

1. Introduction

The simplictic character of the golden mean transfinite numerical system is well known from many fundamental problems in mathematics, physics, chemistry and biology [1,2]. In particular the tiling properties of $\phi = (\sqrt{5} - 1)/2$ is what makes a geometrical figure such as the Penrose pattern drawable by an artist and subsequently used by engineers to produce real life quasi crystals with the once thought forbidden 5-fold symmetries [3]. In fact we could think of a transfinite number such as $= 0.618033989\ldots$ as being made of two parts, a crisp part, namely the rational basic part $0.5 = 1/2$ plus a transfinite fuzzy part, namely $k/10 = 0.01833989\ldots$ We call it fuzzy because we could never write it exactly in the decimal system nor as a ratio of any two integers. The situation is somewhat reminiscent of the concept of a naked and a dressed elementary particle and it is in the meantime well understood that the irrational fuzzy parts of our golden mean is what makes our tiling fit seamlessly by slipping into the gaps between the two different tiles used [4]. Main stream high energy physicists seem to have just started to realize the immensely important role which transfinite numbers, in particular the golden mean, plays in the nonlinear dynamics of particle interactions [5]. It is our intention to illustrate here this importance using a specific example.

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2. The renormalization group for gauge couplings, GUT unification and transfinite harmonization

2.1. Gauge coupling unification

It is well known that when using the standard model low energy coupling it is possible to check the concepts of unification, such as the SU(5) theory [6]. Thus we start at the one loop level of the renormalization approach and compute the beta functions of SU(3), SU(2) and U(1) to the lowest order. Proceeding that way one finds, after some mathematical manipulation and under the tacit assumptions of quantum field theory that

$$\ell\mathrm{n}\frac{M_{\mathrm{u}}}{M_z} = A_1[\bar{\alpha}_{\mathrm{ew}} - A_2\bar{\alpha}3]_{Z_0}$$

and

$$\bar{\alpha}_{\mathrm{u}} = \bar{\alpha}_3 + A_3\ell\mathrm{n}\frac{M_{\mathrm{u}}}{M_z}.$$

Here M_{u} is the grand unification mass, M_z is the mass of Z^0 of the electroweak, $\bar{\alpha}$ is the inverse electromagnetic constant at the Z^0 scale and $\bar{\alpha}_3$ is the inverse strong coupling where a subscript Z^0 means evaluated at $m(Z^0)$. The A_i constants on the other hand stand for the following [6]:

$$A_1 = (6\pi)/[11(1+3C^2)],$$
$$A_2 = 1 + C^2,$$
$$A_3 = b_3/(2\pi),$$
$$C^2 = 5/3,$$
$$b_3 = 11 - (4/3)N_{\mathrm{F}} \quad \text{for non-super symmetric theory,}$$
$$b_3 = 9 - 2N_{\mathrm{F}} \quad \text{for super symmetric theory.}$$

Here N_{F} is the number of generations which is taken to be $N_{\mathrm{F}} = 3$.

In addition we have our experimental data of the standard model [6]

$$\bar{\alpha}_3(M_z) \simeq 8.5,$$
$$\mathrm{Sin}\theta_{\mathrm{w}}(M_z) \simeq 0.23117,$$
$$\bar{\alpha}_{\mathrm{ew}}(M_z) \simeq 127.943,$$
$$M_z = 91 \text{ Gev.}$$

Inserting in our two main formulas, one finds that

$$\ell\mathrm{n}\frac{M_{\mathrm{u}}}{M_z} \cong 30,$$

which means that the unification scale is given by

$$M_{\mathrm{u}} \simeq (10)^{15} \text{ Gev.}$$

Consequently our inverse non-super symmetric quantum gravity coupling constant is

$$\bar{\alpha}_{\mathrm{u}} \cong 42.$$

The exact expression of *E*-infinity is $\bar{\alpha}_{\mathrm{g}} = 42 + 2k = 42.236067977$ [2].

2.2. Transfinite harmonization

Now our transfinite harmonization methodology requires that we carefully evaluate every part of the used expression numerically and then try to find out or even guess the corresponding exact transfinite value. The procedure is much easier to apply in practice than it may sound and the reader must do this independently to see for himself that it is truly simple and in fact, it is partially a great deal of fun to see how familiar connections pop up from nowhere simplifying a seemingly hopelessly complicated expression. Let us start without much ado: From A_1 we see that

$$A_1 = (6\pi)/[11(1+3(5/3))] = (18.84955592)/[11(6)] = \frac{18.8495592}{66} = 0.285599332.$$

In addition we know from the unwritten dictionary of E-infinity that [2,5]

$$6 \to 6 + k = 6.18033989$$

and

$$\pi \to 3 + \phi^3 = 3.236067977.$$

Consequently

$$(6)(\pi) \to (6 + k)(3 + \phi^3) = 20.$$

Now 66 seems a little odd number for E-infinity and the nearest which comes to mind is

$$2(32 + 2k) = 64 + 4k = 64.72135954.$$

Assuming this to be correct, and we will see shortly that it is correct, then A_1 is given by a simple exact expression

$$A_1 = \frac{20}{64 + 2k} = (\phi/2),$$

where ϕ is the golden mean. Next we know from E-infinity that $\bar{\alpha}_{\text{ew}}$ is given exactly by [2,5]

$$\bar{\alpha}_{\text{ew}} = \bar{\alpha}_0 - \bar{\alpha}_3 = (137 + k_0) - 9 = 128 + k_0 = 128.082039325,$$

where $k_0 = \phi^5(1 - \phi^5)$. Furthermore, we have

$$A_2 = 1 + 5/3 = 2.6666,$$

which must be replaced by $(1/\phi)^2$. Consequently we have

$$1 + (5/3) = 2.6666 \to 2 + \phi = (1/\phi)^2 = 2.618033989.$$

Thus our factor is $(A_2)\ (\bar{\alpha}_3) = (1/\phi)(9)$ amounts to 23.5623059. Therefore

$$(1 + C2)(\bar{\alpha}_3) \to (2 + \phi)(9) = 23.5623059.$$

Inserting in $\ell\text{n}\frac{M_{\text{u}}}{M_z}$ one finds

$$\ell\text{n}\frac{M_{\text{u}}}{M_z} = (\phi/2)[128 + k_0 - (2 + \phi)(9)] = (\phi/2)[104.5197334].$$

This is a particularly interesting result because

$$\bar{\alpha}_0|_{\text{GUT}} = (10)(1/\phi)^5 - (6 + k) = 110.9016995 - 6.1803398 = 104.7213596,$$

which is quite close to the used value and we suspect therefore that the correct expression should be

$$\ell\text{n}\frac{M_{\text{u}}}{M_z} = (\phi/2)[\bar{\alpha}_0|_{\text{GUT}}] = (\phi/2)(104.7213596) = \frac{64 + 4k}{2} = 32 + 2k.$$

Amazingly the long and somewhat mystifying expression for $\ell\text{n}\frac{M_{\text{u}}}{M_z}$ boils down to a simple golden mean scaling of the electromagnetic fine structure constant because [2,5]

$$(\bar{\alpha}_0)(\phi)^3 = (137 + k_0)(\phi^3) = 32 + 2k.$$

In other words, $\ell\text{n}\frac{M_{\text{u}}}{M_z}$ is nothing but

$$\ell\text{n}\frac{M_{\text{u}}}{M_z} = (\bar{\alpha}_0)(\phi)^3.$$

Determining the non-super symmetric and the super symmetric coupling is then our next task. To do that we still need A_3 and $\bar{\alpha}_3$. For non-super symmetric theory this is

$$A_3 = b_3/2\pi = \frac{11 - 4}{2\pi} = 7/2\pi = 1.114 \simeq 1.$$

In the case of super symmetric theory on the other hand, one finds

$$A_3 = b_3/2\pi = \frac{3}{2\pi} = 0.477464829 \simeq 1/2.$$

As for $A_3 = 9$, this did not lead us to the correct result for $\ell\text{n}\frac{M_{\text{u}}}{M_z}$ and we know from E-infinity that is should be in general revised to $\bar{\alpha}_3 + \bar{\alpha}_4 = 10$ where $\bar{\alpha}_4 = \bar{\alpha}_{\text{QG}} = 1$ is the coupling of the Planck mass $m_{\text{pl}} \simeq (10^{19})$ Gev.

Inserting in $\bar{\alpha}_0$ one finds

$$\bar{\alpha}_0 = (\bar{\alpha}_3 + 1) + A_3 \ell n \frac{M_u}{M_z} = (\bar{\alpha}_3 + 1) + (1)(32 + 2k) = 10 + 32 + 2k = 42 + 2k = \bar{\alpha}_g,$$

which is the exact E-infinity value. On the other hand for $\bar{\alpha}_{gs}$ with to Higgs multiplets, our factor is $A_3 = 1/2$ and one finds

$$\bar{\alpha}_u = (\bar{\alpha}_3 + 1) + \left(\frac{1}{(2)}\right)(32 + 2k) = (10) + (16 + k) = 26 + k = \bar{\alpha}_{gs}.$$

Again this is the exact E-infinity value [2,5].

3. The universe as a Planck mass and Newton's gravitational constant

From the preceding analysis we see that rather than taking the detour de force to find $\bar{\alpha}_u$ using renormalization groups and ℓn of the ratio M_u/M_z, we could go directly to $|E_8E_8| = 496$ divided by $|\text{SU(3) SU(2) U(1)}| = 12$ to obtain an estimate for the coupling, namely [2,5]

$$\bar{\alpha}_g = 496/12 = 41.666 \simeq 42,$$

which is not very far off the exact value any way. In fact, because fractals are by their very nature scale invariant, and thus in a sense E-infinity is gauge invariant, we could obtain the exact value by transfinite harmonization once we notice that [2,5]

$$|E_8E_8| = 496 \to 496 - k^2$$

and

$$|\text{SU(3)SU(2)U(1)}| = 12 \to 12 - 2\phi^6 = \sqrt{\bar{\alpha}_0}.$$

Consequently we have

$$\bar{\alpha}_g = \frac{496 - k^2}{\sqrt{\bar{\alpha}_0}} = \frac{496 - k^2}{11.7082039325} = 42.3606799,$$

which is the exact E-infinity result.

We could use the preceding analysis to determine Newton's dimensionless gravitational constant $\bar{\alpha}_{NG} \simeq 10^{38}$. The chain of conceptional thoughts is as follows: We know from Witten's T duality that at ultra high energy we meet again the low energy regime and from black hole theories we know that not only elementary particles may be mini black holes, but the universe as a whole may also be regarded as a black hole. At the same time, the universe becomes simplest at the Planck energy scale when the coupling constant of the Planck masses is $\bar{\alpha}_{QG} = 1$. Thus regarding the Planck mass to be formed by $N = (10)^{19}$ Gev/m$_p$ particles where $m_p = 0.939$ Gev is the mass of the protons, then we see that $N \simeq (10)^{19}$ plays the same role of $|E_8E_8| = 496$ massless gauge bosons of Heterotic super string theory. Thus in analogy to

$$\bar{\alpha}_g = \frac{496 - k^2}{\sqrt{\bar{\alpha}_0}},$$

we can write

$$\bar{\alpha}_{QG} = \frac{(10)^{19}}{\sqrt{\bar{\alpha}_{NG}}}.$$

Since $\bar{\alpha}_{QG} = 1$, one finds that

$$\sqrt{\bar{\alpha}_{NG}} = (10)^{19}$$

and therefore

$$\bar{\alpha}_{NG} \cong (10)^{38}$$

which is Newton's dimensionless gravitational constant.

4. Discussion and conclusions

In Table 1 we give a somewhat detailed dictionary of the quantum field theoretical expressions and their transfinitely harmonized counterparts. As a result of this dictionary we can deduce $\bar{\alpha}_{NG} \cong (10)^{38}$ as well as $\bar{\alpha}_g = 42 + 2k$ and $\bar{\alpha}_{gs} = 26 + k$. We could have reached the same results using various somewhat elitarian advanced topology connected to certain exceptional Lie symmetry groups hierarchy and exotic spheres kissing problems in 128 dimensional spaces. To give the reader only the flavour of this subject, let us consider the following sphere kissing problem in 128 dimensions corresponding to $\bar{\alpha}_{ew} \cong 128$. For lattice and non-lattice packing, the kissing numbers are found to be 218044170240 and 88633586495104, respectively [7]. Regarding these numbers as the intersection of super symmetric states of eight groupings, then it is clear that the geometrical average will give us a unification coupling constant. In other words we have for a lattice structure

$$\sqrt[8]{218044170240} = 26.1407 \simeq \bar{\alpha}_{gs}.$$

For a non-lattice structure on the other hand, one finds

$$\sqrt{8863586495104} = 41.5383 \simeq 42 = \bar{\alpha}_g.$$

We see that the lattice structure gives us the large coupling, namely $\bar{\alpha}_{gs} = 26.14$ which is very close indeed to the exact value $\bar{\alpha}_{gs} = 26.18033$.

There are many other amazing connections, for instance between $\bar{\alpha}_0 \simeq 137$ and the exceptional Lie symmetry groups hierarchy. Let us consider again the sphere kissing problem but from the view point of hierarchal structure. Denoting the kissing number by $K(n)$ where n is the corresponding dimension, we see that [7]

$$\sum_0^5 K(n) = 0 + 2 + 6 + 12 + 24 + 40 = 1/4[\mathrm{SL}(2,7)] = 84, \quad \sum_0^6 K(n) = 84 + 72 = |E_6 E_6| = 156$$

and

$$\sum_0^8 K(n) = 522.$$

Consequently embedding $\sum_0^8 K(n)$ in $D^{(26)} = 26$ one finds

$$\sum_0^8 K(n) + D^{(26)} = 522 + 26 = 548 = (4)(137) = (4)(\bar{\alpha}_0),$$

where $\bar{\alpha}_0$ is the inverse electromagnetic fine structure constant.

It is interesting to note that the 576 states leading to the $576/8 = 72$ elementary particles predicted by the Slovenian scientist, L. Marek-Crnjac could be obtained by adding $|D_4| = 28$ group to our hierarchy because $548 + 28 = 576$. This result is particularly interesting for a possible deep connection between the standard model and Freudental's magic square as well as Cvitanovic's magic triangle [7].

We conclude by restating our conviction that nonlinear dynamics, chaos and fractals will revolutionize the way we think about our high energy physics problems [5].

Table 1
A dictionary for E-infinity transfinite simplictic harmonization

Conventional quantum field theory	Corresponding transfinitely harmonized expression
$\ell n \frac{M_u}{M_z}$	$(\bar{\alpha}_0)(\phi)^3 = 32 + 2k$
$\bar{\alpha}_3(M_z)$	$\bar{\alpha}_3 + \bar{\alpha}_4 = 10$
$\frac{6\pi}{11(1+3C^2)}$	$\frac{20}{64+4k} = \phi/2$
$\bar{\alpha}_{ew}(M_z)$	$128 + k_0 = 128.0820393$
$1 + C^2 = 1 + 3/5$	$1 + (1/\phi)^2 = 2 + \phi = 2.618033$
$b_3/(2\pi)$	1 (for non-super symmetric theory)
$b_3/2\pi$	1/2 (for super symmetric theory)
$\bar{\alpha}_u \simeq 40 \sim 42$ (non-super symmetric)	$\bar{\alpha}_g = (\bar{\alpha}_3 + \bar{\alpha}_4) + (\bar{\alpha}_0)(\phi^3) = 10 + 32 + 2k = 42 + 2k$
$\bar{\alpha}_u \simeq 24 \sim 26$ (super symmetric)	$\bar{\alpha}_{gs} = 10 + 1/2(\bar{\alpha}_0)(\phi^3) = 26 + k$

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